Solutions of AR (1) Equations

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A time series is an ordered sequence of values of variables at equally spaced time intervals. Time series models are most often used to obtain an understanding of the underlying factors that produce observed data, then fit these models to forecast and monitor. Time series analysis is used in applications such as budgetary analysis, census analysis, economic forecasting, inventory studies, process and quality control, stock market analysis, utility studies, workload projections, and yield projections.

The most commonly used time series equation is the autoregressive process. The autoregressive process is a difference equation determined by random variables. The distribution of such random variables is the key component in modeling time series. The time series considered in this project is the first order autoregressive equation, written as AR(1). The AR(1) equation is a standard linear difference equation: \( X_t = \mu X_{t-1} + \varepsilon_t, t = 0, \pm 1, \pm 2, \ldots \) where the \( \varepsilon_t \) are called the error terms or innovations and are what make the variability in the time series.

For practical reasons, it is desirable to have a unique solution that is independent of time (stationary) and a function of the past error terms. A solution that is independent of time allows one to be able to avoid an initial condition, which may be difficult to find or at an inconvenient location in a time series. A solution as a function of the past error terms is necessary in models used to forecast.

What conditions are necessary in order to guarantee the existence of a unique stationary solution of the past error terms? The literature typically assumes the error terms are an uncorrelated sequence of random variables with a probability distribution that has zero for the mean and a finite variance. This project aims to explore if weaker assumptions can be made about the error terms and still guarantee the existence of a stationary solution. Aside from the mathematical significance of this, there are practical advantages. Some time series, such as stock market prices, exhibit behavior that does not appear to \( \varepsilon_t \), a model that assumes the conditions of the literature. It is shown that weaker conditions about the error terms can be made, which can still guarantee the existence of the desired solution. These less restrictive conditions allow models to have distributions that are a better fit for time series such as stock market prices.